TRINITY COLLEGE

Year 12 Specialist Mathematics Units 3,4
Test 1 2019

Section 1 Calculator Free

Complex Numbers, Functions

STUDENT'S NAME _____

DATE: Wednesday 6th March **TIME:** 50 minutes **MARKS**: 53

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, one A4 page of notes

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (3 marks)

Given $f(x) = \sqrt{x} + 2$ and $g(x) = x^2 - 1$

Determine the domain and range of $y = f(g(x))$

2. (7 marks)

For the expression $2z^4 - z^3 + 13z^2 - 4z + 20$

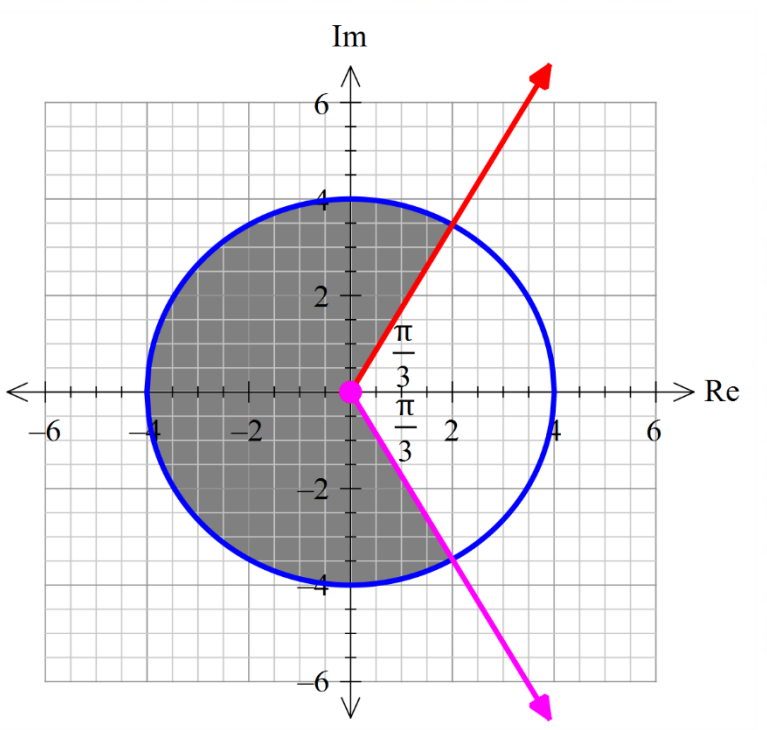
(a) show $z - 2i$ is a factor of the expression [2]

(b) state another factor of the expression [1]

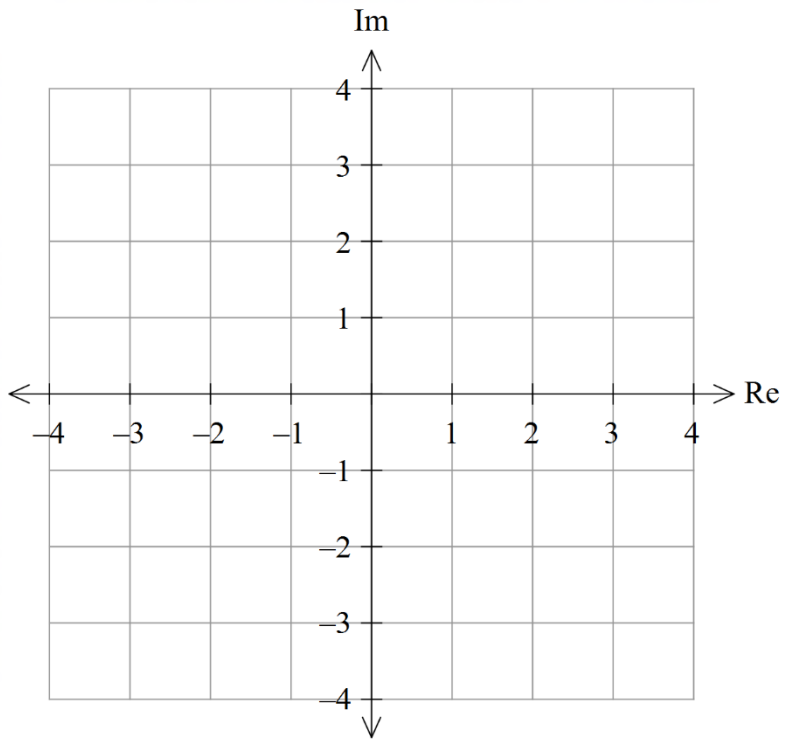
(c) hence solve $2z^4 - z^3 + 13z^2 - 4z + 20 = 0$ [4]

3. (8 marks)

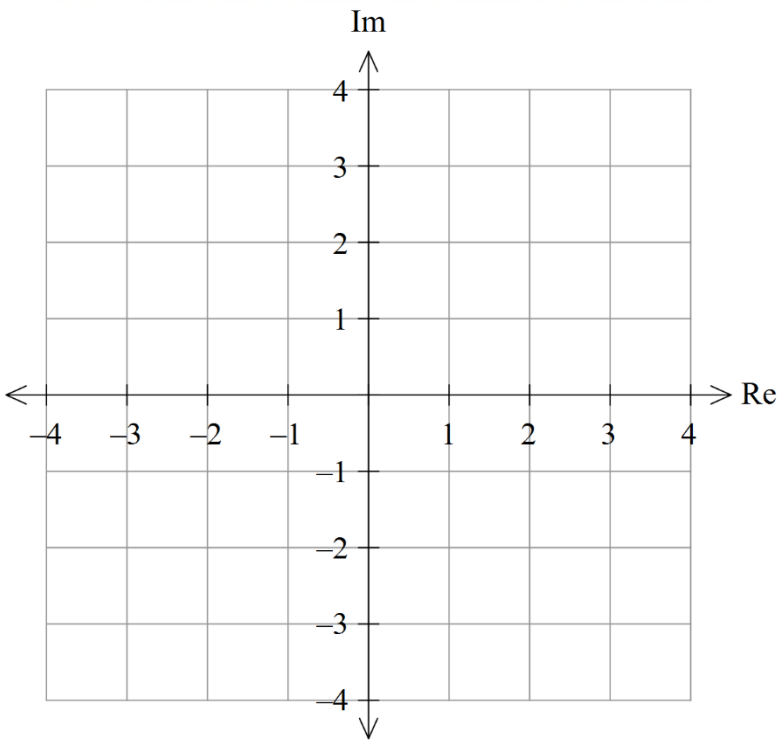
(a) Describe fully the shaded region show. [3]



(b) Sketch $1 < |z-(i)| \leq 2 \quad \cap \quad -\pi \leq Arg(z) \leq 0$ [3]



(c) Sketch $Arg(z) + Arg(2 + i\sqrt{2}) = 0$ [2]

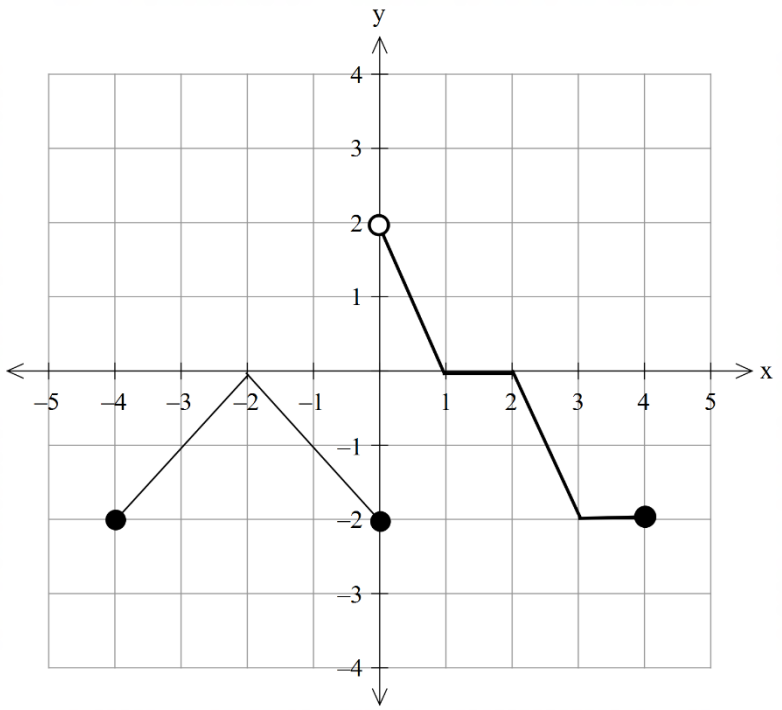


4. (6 marks)

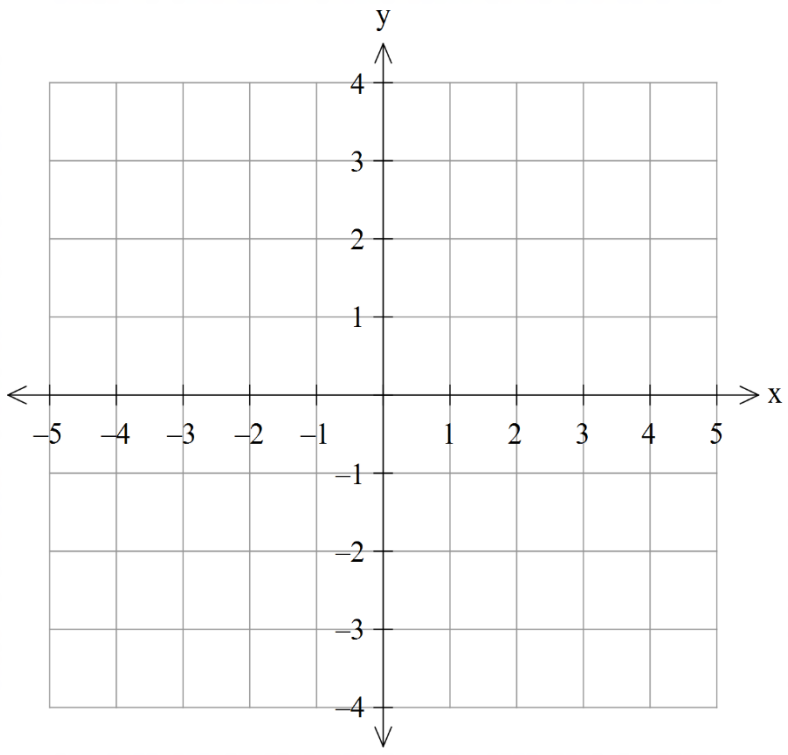
Solve $z^5 = \frac{-i}{32}$. Answer may be given in polar form.

5. (7 marks)

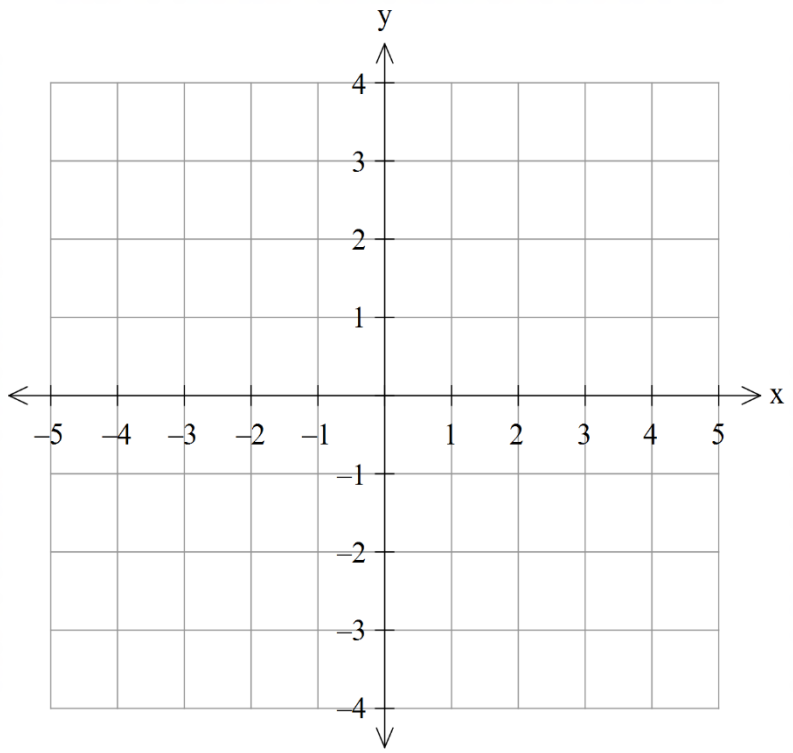
Given $y = f(x)$ as shown below



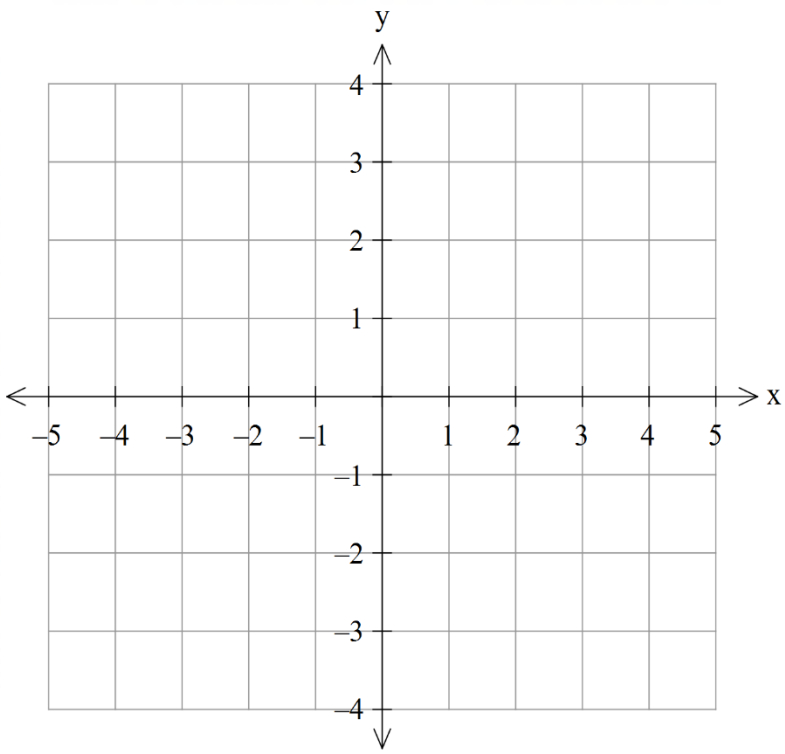
(a) sketch $y = -f|x|$ [2]



(b) sketch $y = \dfrac{1}{f(x)}$ [2]

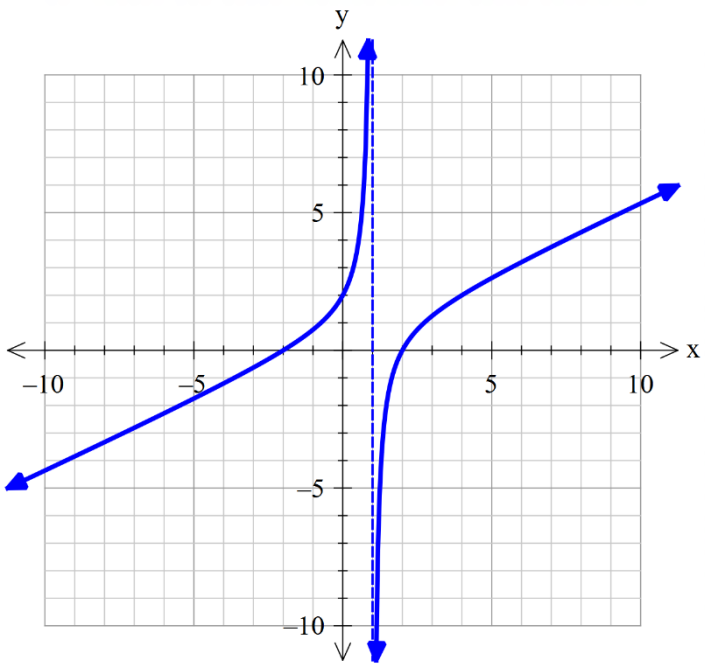


(c) solve $\left|f(x)+1\right| = 2$ [3]



6. (8 marks)

The graph below shows the function $f(x)=\dfrac{ax^2+b}{2x+c}$.



(a) Determine the value of a, b and c. [3]

(b) The function can also be written in the form of $f(x)=px+q+\dfrac{r}{2x+c}$. Determine the values of p, q and r. [3]

(c) State the equations of all asymptotes. [2]

7. (6 marks)

Given $f(x) = x^2 - 1$ where x real and $g(x) = \sqrt{9 - x^2}$ where $-3 \le x \le 3$

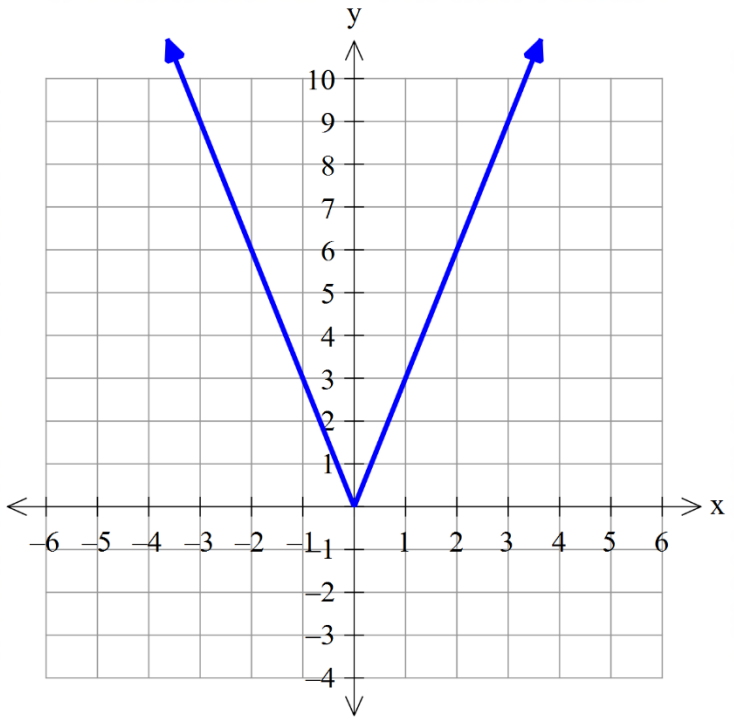
(a) determine an expression for $f(g(x))$ and state its domain and range [3]

(b) determine $h^{-1}(x)$ where $h(x) = f(g(x))$, $-2 \le x \le 0$ [2]

(c) state the range of $h^{-1}(x)$ [1]

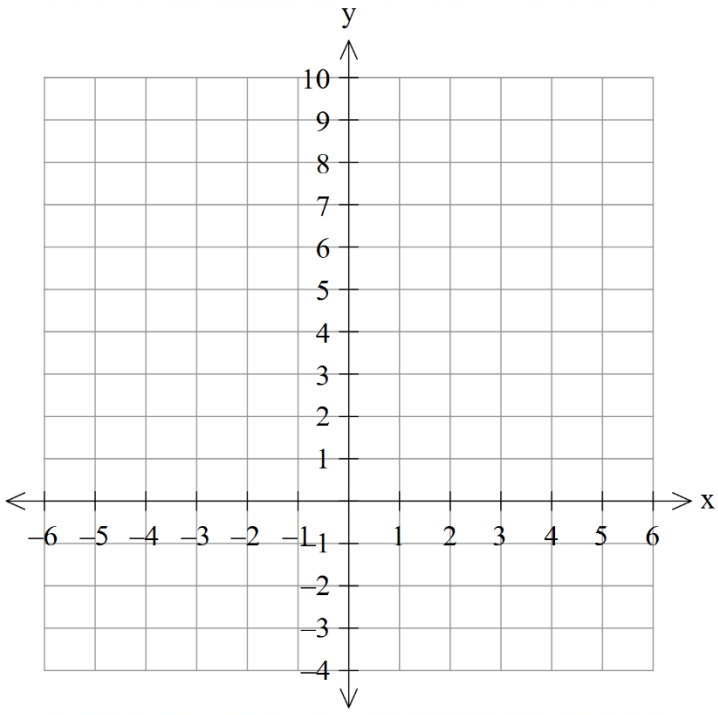
8. (8 marks)

The graph of $y = |3x|$ is drawn on the axes below.



(a) sketch $y = |x-4|-2$ on the axes above. [2]

(b) sketch $y = |3x|+|x-4|-2$ on the axes below. [3]



(c) hence solve $|3x|+|x-4| \le 10$ [3]